Romanian Masters In Mathematics 2008

Bucharest

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1 Let ABC be an equilateral triangle and P in its interior. The distances from P to the triangle's sides are denoted by a^2, b^2, c^2 respectively, where $a, b, c > 0$. Find the locus of the points P for which a, b, c can be the sides of a non-degenerate triangle.

2 Prove that every bijective function $f : \mathbb{Z} \to \mathbb{Z}$ can be written in the way $f = u + v$ where $u, v : \mathbb{Z} \to \mathbb{Z}$ are bijective functions.

3 Let $a > 1$ be a positive integer. Prove that every non-zero positive integer N has a multiple in the sequence $(a_n)_{n\geq 1}$, $a_n = \left\lfloor \frac{a^n}{n} \right\rfloor$.

4 Consider a square of sidelength n and $(n+1)^2$ interior points. Prove that we can choose 3 of these points so that they determine a triangle (eventually degenerated) of area at most $\frac{1}{2}$.