SUBIECTUL II (7p)

a) Fie I centrul cercului înscris în triunghiul ABC. Arătați că $\overrightarrow{AI}=\dfrac{b\overrightarrow{AB}+c\overrightarrow{AC}}{a+b+c}$, unde $a=BC,\ b=AC,\ c=AB.$

b) În triunghiul ABC se consideră bisectoarele AA', BB', CC'. Dacă $\overrightarrow{AA'}+\overrightarrow{BB'}+\overrightarrow{CC'}=\vec{0}$, arătați că triunghiul ABC este echilateral.

Rezolvare:

a) Aplicand teorema bisectoarei in triunghiurile $AA'C$ si ABC obtinem $\dfrac{AI}{IA'}=\dfrac{b}{A'C}$ si

$\dfrac{BA'}{c}=\dfrac{A'C}{b}=\dfrac{a}{b+c}$, deci $\dfrac{AI}{AA'}=\dfrac{b+c}{a+b+c}$ .... 1 p

$\overrightarrow{AI}=\dfrac{b+c}{a+b+c}\overrightarrow{AA'}$ 1p

$\dfrac{BA'}{A'C}=\dfrac{c}{b}\Rightarrow \overrightarrow{AA'}=\dfrac{\overrightarrow{AB}+\frac{c}{b}\overrightarrow{AC}}{1+\frac{c}{b}}=\dfrac{b\overrightarrow{AB}+c\overrightarrow{AC}}{b+c}$ 1p

$\overrightarrow{AI}=\dfrac{b\overrightarrow{AB}+c\overrightarrow{AC}}{a+b+c}$ 1p

b) Folosind a) obtinem

$\overrightarrow{AB}\left(\dfrac{b}{b+c}-\dfrac{a}{a+c}\right)+\overrightarrow{BC}\left(\dfrac{c}{a+c}-\dfrac{b}{a+b}\right)+\overrightarrow{AC}\left(\dfrac{c}{b+c}-\dfrac{a}{a+b}\right)=\vec{0}$ 1p

$\overrightarrow{AB}\left(1-\dfrac{a}{a+c}-\dfrac{a}{a+b}\right)+\overrightarrow{BC}\left(\dfrac{c}{a+c}+\dfrac{c}{b+c}-1\right)=\vec{0}$ si $\overrightarrow{AB},\overrightarrow{BC}$ necoliniari implica

$1-\dfrac{a}{a+c}-\dfrac{a}{a+b}=\dfrac{c}{a+c}+\dfrac{c}{b+c}-1=0$ 1p

$a^2=bc,\ c^2=ab,$ deci $a=b=c$ 1p