SUBIECTUL II

Fie $A, B \in \mathrm{M}_2(\square)$ astfel incat AB=BA, $\det(A) = -3$ si $\det\left(A + \sqrt{3}B\right) = 0$.

Sa se calculeze $\det\left(A^2 + B^2 - AB\right)$.

G.M. 12/2011.

Barem de corectare:

1. $\det(xA + yB) = x^2 \det(A) + xy\Delta + y^2 \det(B)$ 2p

2. Aplicam 1. si obtinem $\Delta = 0$ si $\det(B) = 1$ 1p

3. Descompunem $A^2 + B^2 - AB = (A - \alpha B)(A - \beta B)$ unde α, β sunt solutiile ecuatiei $x^2 - x + 1 = 0$ 1p

4. Calculam, folosind 1., $\det(A - \alpha B); \det(A - \beta B)$ 2p

5. Finalizare $\det\left(A^2 + B^2 - AB\right) = 13$ 1p

SUBIECTUL III

Se considera sirul $(x_n)_{n\geq 1}$ dat prin $x_1 = 5$ si $x_{n+1} = x_n^2 - 2, \forall n \geq 1$.

Calculati

$$\lim_{n\to\infty} \frac{x_1 x_2 \ldots x_n}{x_{n+1}}.$$

Barem de corectare:

1. Notand $y_n = x_n^2$ obtinem $y_{n+1} - 4 = y_n(y_n - 4)$ 2p

2. Ciclarea relatiei de la1 .... 1p

3. Obtinerea relatiei $\frac{y_1 y_2 \ldots y_n}{y_{n+1}} = \frac{y_{n+1} - 4}{21 y_{n+1}}$ 2p

4. $\lim_{n\to\infty} y_n = \infty$ 1p

5. $\lim_{n\to\infty} \frac{y_1 y_2 \ldots y_n}{y_{n+1}} = \frac{1}{21}$ si finalizare 1p