

Clasa a VII-a

Rezolvare:

1. O problema cunoscuta din manual, punctata astfel:

- Figura corespunzatoare.....1p
a) BEDF paralelogram (diagonalele se injumatatesc).....2p
b) Practic probabil este parte la punctul a).....2p
c) [AO] mediana in triunghiul ABD; $EO = \frac{1}{3} AO$, $\Rightarrow E$ centrul de greutate al triunghiului ABD.....2p

2. a) $\frac{1}{2} \left(\frac{1}{2^n + 1} - \frac{1}{2^{n+1} + 1} \right) = \frac{1}{2} \cdot \frac{2^{n+1} + 1 - 2^n - 1}{(2^n + 1) \cdot (2^{n+1} + 1)} = \frac{2^{n-1}}{(2^n + 1) \cdot (2^{n+1} + 1)}$ 3p

b)

$$\frac{1}{3 \cdot 5} + \frac{2}{5 \cdot 9} + \frac{4}{9 \cdot 17} + \dots + \frac{2^{n-1}}{(2^n + 1) \cdot (2^{n+1} + 1)} = \frac{1}{2} \cdot \left(\frac{1}{3} - \frac{1}{5} + \frac{1}{5} - \frac{1}{9} + \dots + \frac{1}{2^n + 1} - \frac{1}{2^{n+1} + 1} \right) =$$

$$= \frac{1}{2} \cdot \left(\frac{1}{3} - \frac{1}{2^{n+1} + 1} \right) = \frac{2^n - 1}{3 \cdot (2^{n+1} + 1)}; \Rightarrow n = 2010$$
 4p

3. Figura corespunzatoare1p

Fie punctele E,F,K, astfel incat:

$$E \in (AD), F \in (BC), DE = \frac{1}{3} AD, CF = \frac{1}{3} BC, K \text{ mijlocul}$$

lui[EF].....2p

Aplicam proprietatea: mijloacele laturilor unui patrulater convex determina un paralelogram,.....1p

pentru patrulaterul ABFE, si CDQN obtinem ca patrulaterul MNKQ, EPFT sunt paralelograme cu semidiagonala [TK]

comuna.....2p

Diagonalele [MK] si [TP] au ca dreapta suport TK.

Deci M,T,P sunt coliniare.....1p

4. a) $\frac{1}{k} - \frac{1}{k+d} = \frac{k+d-d}{k \cdot (k+d)} = \frac{1}{k \cdot (k+d)}$ 3p

b) $1 - \frac{2}{1 \cdot (1+2)} - \frac{3}{(1+2) \cdot (1+2+3)} - \dots - \frac{100}{(1+2+\dots+99) \cdot (1+2+3+\dots+100)} =$
 $= 1 - 1 + \frac{1}{3} - \frac{1}{3} + \frac{1}{6} - \frac{1}{6} + \frac{1}{10} - \dots - \frac{1}{1+2+\dots+98} + \frac{1}{1+2+\dots+99} - \frac{1}{1+2+3+\dots+99} +$
 $\frac{1}{1+2+\dots+99} - \frac{1}{1+2+\dots+99+100} = \frac{1}{1+2+3+\dots+99+100} = \frac{1}{\frac{100 \cdot 101}{2}} = \frac{2}{100 \cdot 101} < \frac{2}{100^2} = 0,0002$

2p