BAREM CLASA A –XII-A

1. Considerăm funcţia $g:(0,\infty)\to R,\ \ g(x)=\dfrac{f(x)}{x^k\cdot e^{px}}-b\ln x$ ……………………..3p

Avem $g'(x)=\dfrac{f'(x)\cdot x^k\cdot e^{px}-\left(k\cdot x^{k-1}\cdot e^{px}+p\cdot x^k\cdot e^{px}\right)\cdot f(x)}{x^{2k}\cdot e^{2px}}-\dfrac{b}{x}=$

$=\dfrac{f'(x)-\left(p+\dfrac{k}{x}\right)\cdot f(x)}{x^k\cdot e^{px}}-\dfrac{b}{x}=\dfrac{b\cdot x^{k-1}\cdot e^{px}}{x^k\cdot e^{px}}-\dfrac{b}{x}=0,\ \ \forall x>0$ ………………………..2p

$\Rightarrow g$ este o funcţie constantă pe $(0,\infty)$. Cum $g(1)=\dfrac{f(1)}{e^p}=a\Rightarrow g(x)=a,\ \ \forall x>0\Rightarrow$

$\Rightarrow a=\dfrac{f(x)}{x^k\cdot e^{px}}-b\ln x\Rightarrow f(x)=(a+b\ln x)\cdot x^k\cdot e^{px},\ \ x>0$ ………………………..2p

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Total 7 puncte

2. $I=\int_0^{\frac{\pi}{4}}\dfrac{\cos x}{e^x\left[1+2013\cdot e^{-x}(\cos x-\sin x)\right]}dx$ ………………………………………..1p

$I=\int_0^{\frac{\pi}{4}}\dfrac{e^{-x}\cos x}{1+2013\cdot e^{-x}(\cos x-\sin x)}dx$ …………………………………………….1p

Notăm $t=1+2013\cdot e^{-x}(\cos x-\sin x),\ \ dt=-4026\cdot e^{-x}\cos x dx$ ……………………2p

$\Rightarrow I=-\dfrac{1}{4026}\int_{2014}^{1}\dfrac{dt}{t}=\dfrac{1}{4026}\int_{1}^{2014}\dfrac{dt}{t}$ ……………………………………………..2p

Finalizare $I=\dfrac{\ln 2014}{4026}$ ……………………………………………………………1p

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Total 7 puncte

3. Notăm cu $H=\{x\in G\mid xa=ax,\ \ \forall a\in A\}$ ……………………………………….1p

Fie $x,y\in H$ şi $a\in A\Rightarrow a(xy)=(ax)y=(xa)y=x(ya)=(xy)a\Rightarrow xy\in H$

Deci H este parte stabilă a lui G în raport cu operaţia din G ………………………..2p

Cum G este finit $\Rightarrow H\le G$ ……………………………………………………………1p

Folosind teorema lui *Lagrange* $\Rightarrow |G|\,\vdots\,|H|\overset{def}{=}h$ ……………………………………..1p

Deci $h\mid n,\ \ h>m,\ \ m\mid n\Rightarrow h=n\Rightarrow H=G$ …………………………………………2p

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Total 7 puncte

4. Dacă $a^2+b^2=ab\Rightarrow a\left(a^2+b^2\right)=a^2b$ şi $\left(a^2+b^2\right)b=ab^2\Rightarrow$

$\Rightarrow a^3+ab^2=a^2b$ şi $a^2b+b^3=ab^2$

Adunând cele două relaţii $\Rightarrow a^3+b^3=0$ (1) ………………………………………3p

Cum $a^2\left(a^2+b^2\right)=a^3b\Rightarrow a^4+a^2b^2=a^3b\overset{(1)}{\Rightarrow}a^2b^2=-a^4-b^4$ (2) …………………..2p

Deoarece $b\left(a^2+b^2\right)a=baba\Rightarrow ba^3+b^3a=(ba)^2\underset{(2)}{\overset{(1)}{\Longrightarrow}}(ba)^2=-a^4-b^4=a^2b^2$ ………1p

Din $(ab)^2=\left(a^2+b^2\right)^2=\left(a^2+b^2\right)\left(a^2+b^2\right)=a^4+a^2b^2+b^2a^2+b^4\overset{(2)}{\Rightarrow}(ab)^2=b^2a^2$ ..1p

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Total 7 puncte