CONCURSUL INTERJUDEŢEAN DE MATEMATICĂ
" PETRU MOROŞAN -TRIDENT "
Ediţia a X-a , Secţiunea B (M_2),
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CLASA a X a

1. a) Demonstraţi identitatea: $\frac{1}{k(k+1)}=\frac{1}{k}-\frac{1}{k+1}, k\in\mathbb{N}^*$

b) Determinaţi produsul rădăcinilor ecuaţiei:

$$\sum_{k=1}^{n}\left(\log_x 2012^{\frac{1}{k}}\right)\cdot\left(\log_x 2012^{\frac{1}{k+1}}\right)+\frac{1}{\left(\log_{2012}x\right)^2}=2-\frac{1}{n+1}, x\in(0,+\infty)-\{1\}.$$

Roxandra Murea

2. Fie $z_1, z_2 z_3\in\mathbb{C}$ distincte,cu $|z_1|=|z_2|=|z_3|=1, |kz_1-z_2-z_3|=|kz_2-z_1-z_3|=|kz_3-z_1-z_2|$, $k\in\mathbb{R}/\{-1\}$, atunci $z_1+z_2+z_3=0$.

Gheorghe Alexe, Brăila

3. Fie $a_1,a_2,...a_n\in(0,1)$ sau $a_1,a_2....a_n\in(1,\infty)$, $n\in\mathbb{N}, n\geq 2$. Demonstraţi inegalităţile:

a) $\log_{a_1} a_2 + 2\cdot\sqrt{\log_{a_2} a_3}+3\cdot\sqrt[3]{\log_{a_3} a_1}\geq 6$;

b) $\log_{a_1} a_2 + 2\cdot\sqrt{\log_{a_2} a_3}+3\cdot\sqrt[3]{\log_{a_3} a_1}+...+n\cdot\sqrt[n]{\log_{a_n} a_1}\geq\frac{n(n+1)}{2}$.;

Runceanu Emilian

Notă: Toate subiectele sunt obligatorii. Timp de lucru 2 ore.