BAREM CLASA AVII-A

1) Ecuatia $\sqrt{x+3}+\sqrt{5-x}=n(n+3)$ $n \in Z$, are solutii numere intregi.
Aflati *n.*

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$x+3 \geq 0$ *si* $5-x \geq 0 \Rightarrow x \in [-3,5]$ **(1 pct)**

$\left.\begin{array}{l} x \in [-3,5] \\ x \in Z \end{array}\right| \Rightarrow X \in \{-3,-2,....4,5\}$ **(1 pct)**

$-3,-2,-1,0,2,3,4,5$ *nu pot fi solutii ale inecuatiei* $\Rightarrow$ **(2 pct)**

$\Rightarrow n(n+3)=4 \Rightarrow n^2+3n-4=0 \Rightarrow n^2-n+4n-4=0 \Rightarrow (n-1)(n+4)=0 \Rightarrow n=1$ *sau* $n=-4$ **(2 pct)**

$\Rightarrow x=1$ solutie **(1 pct)**

2) Pe latura BC a $\triangle$ABC se considera punctele D si E astfel incat BD=DE=EC. Mediana BB' (B' $\in$ AC) intersecteaza pe AD in M, iar mediana CC' (C' $\in$ AB) intersecteaza pe AE in N. Aratati ca :

a. BMNC este trapez

b. MN = $\frac{1}{4}$BC

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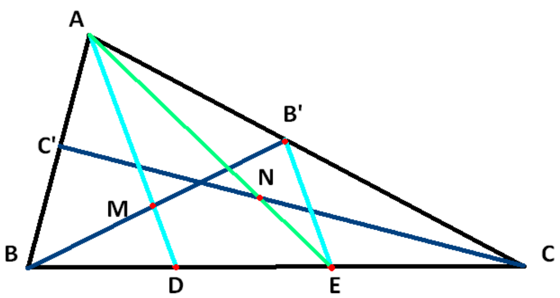

C'B' linie mijlocie in $\triangle$ ABC $\Rightarrow$ C'B' $\parallel$ BC $\Rightarrow$ BCB'C' trapez **(2 pct)**

B'E linie mijlocie in $\triangle$ ADC $\Rightarrow$ B'E $\parallel$ AD ; in $\triangle$ BB'E, D mijlocul lui BE, DM $\parallel$ B'E $\Rightarrow$M mijlocul lui BB'. Analog N mijlocul lui CC'. **(2 pct)**

In trapezul BC'B'C M si N mijloace de diagonale $\Rightarrow$ MN $\parallel$ BC $\parallel$ B'C' $\Rightarrow$

$\Rightarrow$ 1) MNCB trapez **(1 pct)**

2) MN=$\frac{BC-B'C'}{2}=\frac{BC-\frac{BC}{2}}{2}=\frac{BC}{4}$ **(2 pct)**

3) Rezolvati in Z ecuatia : 2xy=3(x+32)

2xy-3x=96 $\Leftrightarrow$ x(2y-3)=96 **(2 pct)**

$$\left.\begin{array}{l} x=\dfrac{96}{2y-3} \\ x\in Z\end{array}\right| \Rightarrow \begin{array}{l}\dfrac{96}{2y-3}\in Z \\ 2Y-3 \quad IMPAR\end{array} \Rightarrow \begin{array}{c} 2Y-3=\pm1 \\ sau \\ 2y-3=\pm3\end{array}$$ **(3 pct)**

Finalizare **(2 pct)**

4) In patrulaterul ABCD punctele M, Q, N, P sunt mijloacele laturilor AB, BC, CD respectiv DA.

Demonstrati ca: $\dfrac{AC+BD}{2} < MN+PQ \leq \dfrac{AB+BC+CD+DA}{2}$

Fie $\{O\}=MN\cap PQ$

$$\left.\begin{array}{l} OM+ON>MQ=\dfrac{AC}{2} \\ OM+OP>PM=\dfrac{DB}{2} \\ OP+ON>PN=\dfrac{AC}{2} \\ ON+OQ>QN=\dfrac{DB}{2} \end{array}\right\} \text{2 pct.}$$

$$\left.\begin{array}{l} 2(OM+ON+OP+OQ)>AC+BD \\ MN+PQ>\dfrac{AC+BD}{2} \end{array}\right\} \text{2 pct.}$$

Se obtine egalitatea daca M,R,N *coliniare* 1 pct.

$\Downarrow$

$$\left.\begin{array}{l} \textit{Fie } R \textit{ mijlocul lui } DB \\ MN<\dfrac{AD}{2}+\dfrac{BC}{2} \end{array}\right\} \text{2 pct.}$$

Analog $PQ<\dfrac{AB}{2}+\dfrac{CD}{2}$ 1 pct.

Finalizare 1 pct