Olimpiada de matematică
Etapa locală, Caraş-Severin, 16.02.2013

Clasa a IX-a, BAREME

Problema 1:

orice exemplu corect ....2 pct

$ab=cd$ ....2 pct

Demonstraţie ....3 pct

Problema 2:

Arată că $\dfrac{x-\frac{1}{x}}{x+\frac{y+z}{x}} \geq \dfrac{x-\frac{1}{x}}{x+y+z}$ pentru $x\geq 1$ şi pentru $x<1$ ....2 pct

Rescrie inegalitatea sub forma $\dfrac{z^2-1}{x+y+z^2}+\dfrac{y^2-1}{x+z+y^2}+\dfrac{x^2-1}{z+y+x^2}\geq 0 \Leftrightarrow \sum \dfrac{x^2-1}{x^2+y+z}\geq 0$. 3pct

$\sum \dfrac{x-\frac{1}{x}}{x+y+z}=\dfrac{1}{x+y+z}\left(x+y+z-\dfrac{1}{x}-\dfrac{1}{y}-\dfrac{1}{z}\right)\geq 0$ ....2 pct

Problema 3:

Obţine pentru orice punct M relaţia $a\overrightarrow{MA}+b\overrightarrow{MB}+c\overrightarrow{MC}=(a+b+c)\overrightarrow{MI}$ 1pct

Obţine $\overrightarrow{GA}=-\frac{2}{3}\overrightarrow{AP}$, $\overrightarrow{GB}=-\frac{2}{3}\overrightarrow{BR}$ şi $\overrightarrow{GC}=-\frac{2}{3}\overrightarrow{CS}$, unde P,R,S mij lat $(BC),(AC),(AB)$ 2pct

Inlocuieşte relaţiile de mai sus şi obţine $(a+b+c)\overrightarrow{GI}=-\frac{2}{3}\left(a\overrightarrow{AP}+b\overrightarrow{BR}+c\overrightarrow{CS}\right)$ 1pct

Foloseşte $2\overrightarrow{AP}=\overrightarrow{AB}+\overrightarrow{AC}$ şi ajunge la $(a+b+c)\overrightarrow{GI}=-\frac{1}{3}\left[\left(2a-b-c\overrightarrow{AB}\right)+\left(b+a-2c\right)\overrightarrow{BC}\right]$ 2pct

$IG \parallel BC \Leftrightarrow \overrightarrow{IG}$ şi $\overrightarrow{BC}$ au aceeaşi direcţie $\Leftrightarrow 2a-b-c=0$ sau $b+c=2a$ 1pct

Problema 4:

$\sum_{k=1}^{n} MA_k=\sum_{k=1}^{n}|\overrightarrow{OA_k}-\overrightarrow{OM}|$ 2pct

$\sum_{k=1}^{n}|\overrightarrow{OA_k}-\overrightarrow{OM}|\cdot|\overrightarrow{OA_k}|\geq \sum_{k=1}^{n}(\overrightarrow{OA_k}-\overrightarrow{OM})\cdot\overrightarrow{OA_k}=\sum_{k=1}^{n}(1-\overrightarrow{OM}\cdot\overrightarrow{OA_k})$ 2pct

$\sum_{k=1}^{n}(1-\overrightarrow{OM}\cdot\overrightarrow{OA_k})=n-\overrightarrow{OM}\cdot\sum_{k=1}^{n}\overrightarrow{OA_k}=n$ 3pct