Ministerul Educației, Cercetării, Tineretului si Sportului
Inspectoratul Şcolar al Judeţului Timiş

CONCURSUL NAȚIONAL DE MATEMATICĂ APLICATĂ
"ADOLF HAIMOVICI"
etapa locală – 9 februarie 2013
CLASA A XI-A
BAREM DE CORECTARE
Filiera teoretică: profil real, specializarea ştiinţe ale naturii

SUBIECTUL I

a) $f(B) = AB - BA = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ **2p**

b) Fie $X = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in M_2(R)$ soluție a ecuației $f(X) = O_2$

$\Rightarrow \begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix} \Leftrightarrow \begin{pmatrix} 2a+3c & 2b+3d \\ a+2c & b+2d \end{pmatrix} = \begin{pmatrix} 2a+b & 3a+2b \\ 2c+d & 3c+2d \end{pmatrix}$ **2p**

Deduce $X = \begin{pmatrix} a & 3c \\ c & a \end{pmatrix} \in M_2(R).$ **1p**

c) Demonstrează prin inducție matematică propoziția $f(X) = O_2 \to f(X^n) = O_2, \forall n \in N^*$ **2p**

SUBIECTUL II

a) $\Delta(0, 1, -1) = \begin{vmatrix} 0 & 1 & 4 \\ 1 & 4 & 9 \\ 1 & 0 & 1 \end{vmatrix}$ **1p**

= -8 **1p**

b) $\Delta(x, y, z) = \begin{vmatrix} x^2 & (x+1)^2 & (x+2)^2 \\ y^2 & (y+1)^2 & (y+2)^2 \\ z^2 & (z+1)^2 & (z+2)^2 \end{vmatrix}$ **1p**

deduce $\Delta(x, y, z) = -4\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = -4(x-y)(y-z)(z-x)$ **2p**

c) $A_{ABC} = \frac{1}{2}\left|\begin{vmatrix} a & (a+1)^2 & 1 \\ a+1 & (a+2)^2 & 1 \\ a+2 & (a+3)^2 & 1 \end{vmatrix}\right|$ **1p**

=1 **1p**

SUBIECTUL III

a) $\lim\limits_{\substack{x\to 2\\ x<2}} f(x)=0$, $\lim\limits_{\substack{x\to 2\\ x>2}} f(x)=a$ şi **1p**

$\exists \lim\limits_{x\to 2} f(x) \Leftrightarrow \lim\limits_{\substack{x\to 2\\ x<2}} f(x)=\lim\limits_{\substack{x\to 2\\ x>2}} f(x)$ şi deduce $a=0$ **1p**

b) $\lim\limits_{\substack{x\to 0\\ x<0}} f(x)=0-\sqrt{e}=-\sqrt{e}$ şi $\lim\limits_{\substack{x\to 0\\ x>0}} f(x)=\infty$ **1p**

$\Rightarrow x=0$ asimptotă verticală . **1p**

c) $\lim\limits_{\substack{x\to 2\\ x<2}} \dfrac{f(x)}{x^2-5x+6}=\lim\limits_{\substack{x\to 2\\ x<2}} \dfrac{e^{\frac{1}{2}}\left(e^{\frac{1}{x}-\frac{1}{2}}-1\right)}{(x-2)(x-3)}$ **1p**

$=\dfrac{\sqrt{e}}{4}$ **1p**

$\lim\limits_{\substack{x\to 2\\ x>2}} \dfrac{f(x)}{x^2-4}=\dfrac{1}{4}$ **1p**

SUBIECTUL IV

a) $D_{\max}=R\setminus\left\{\dfrac{3}{5}\right\} \Rightarrow (3a-1)^2x^2+x+1\ge 0,\ \forall x\in R\setminus\left\{\dfrac{3}{5}\right\} \Leftrightarrow \Delta\le 0$ **1p**

Finalizare $a\in\left(-\infty,\dfrac{1}{6}\right]\cup\left[\dfrac{1}{2},\infty\right)$. **1p**

b) $\lim\limits_{x\to-\infty} f(x)=\lim\limits_{x\to-\infty}\dfrac{\sqrt{x^2\left[(3a-1)^2+\dfrac{1}{x}+\dfrac{1}{x^2}\right]}}{x\left(5-\dfrac{3}{x}\right)}=\dfrac{-|3a-1|}{5}$ **1p**

$=\lim\limits_{x\to-\infty}\dfrac{-x\sqrt{\left[(3a-1)^2+\dfrac{1}{x}+\dfrac{1}{x^2}\right]}}{x\left(5-\dfrac{3}{x}\right)}$ **1p**

$=\dfrac{-|3a-1|}{5}$ **1p**

Deduce $a_1=-3\in\left(-\infty,\dfrac{1}{6}\right]\cup\left[\dfrac{1}{2},\infty\right)$. , $a_2=\dfrac{11}{3}\in\left(-\infty,\dfrac{1}{6}\right]\cup\left[\dfrac{1}{2},\infty\right)$. Deci $a\in\left\{-3;\dfrac{11}{3}\right\}$. **2p**