



**Concursul Județean „DAN BARBILIAN” – Ediția a XXVI-a
CURTEA DE ARGEȘ - 24 noiembrie 2012**

Clasa a X-a Varianta 2

BAREM de CORECTARE și NOTARE:

1. $xy+xz+zx=\frac{1}{2}[(x+y+z)^2 - (x^2 + y^2 + z^2)]$2 p

Inegalitatea ceruta este echivalenta cu :

$2(\sqrt{x} + \sqrt{y} + \sqrt{z}) + x^2 + y^2 + z^2 \geq 9$ 1 p

Se aplica inegalitatea mediilor.....3 p

$$x^2 + 2\sqrt{x} = x^2 + \sqrt{x} + \sqrt{x} \geq 3x$$

$$y^2 + 2\sqrt{y} \geq 3y$$

$$z^2 + 2\sqrt{z} \geq 3z$$

Finalizare.....1 p

2. Inegalitatea mediilor :

$m \leq \sqrt[n]{a_1 a_2 \dots a_n}$ 2 p

$\log_{a_1} m \geq \frac{1}{n} (\log_{a_1} a_1 + \log_{a_1} a_2 + \dots + \log_{a_1} a_n)$ 2 p

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$\log_{a_n} m \geq \frac{1}{n} (\log_{a_n} a_1 + \log_{a_n} a_2 + \dots + \log_{a_n} a_n)$

$\log_{a_i} a_j > 0 \Rightarrow \log_{a_i} a_j + \log_{a_j} a_i \geq 2, \forall i, j \in \{1, 2, \dots, n\}$ 1 p

Finalizare.....2 p

3. Notam $a + b = x$

$$b + c = y$$

$$c + a = z$$

$$\Rightarrow a = \frac{x-y+z}{2}, b = \frac{x+y-z}{2}, c = \frac{-x+y+z}{2} \dots\dots\dots 2 \text{ p}$$

Daca $x = y \Rightarrow a = c$ fals! $\Rightarrow x \neq y$

Analog $x \neq z, y \neq z \dots\dots\dots 1 \text{ p}$

Din $y^3 = x^3$ si $z^3 = x^3 \Rightarrow y = \varepsilon x, z = \varepsilon^2 x, \varepsilon^3 = 1, \varepsilon^2 + \varepsilon + 1 = 0 \dots\dots\dots 2 \text{ p}$

Se calculeaza $a^3 = \left(\frac{x-y+z}{2}\right)^3 = \frac{x^3}{8} (1 - \varepsilon + \varepsilon^2)^3 = -x^3 \dots\dots\dots 0,5 \text{ p}$

Analog $b^3 = -x^3 \dots\dots\dots 0,5 \text{ p}$

$c^3 = -x^3 \dots\dots\dots 0,5 \text{ p}$

Finalizare $\dots\dots\dots 0,5 \text{ p}$

4. $\lg 2^{2009} = 2009 \lg 2 = 604,7.. \Rightarrow 2^{2009}$ are 605 cifre $\dots\dots\dots 1 \text{ p}$

$\lg 2^{2010} = 2010 \lg 2 = 605,01.. \Rightarrow 2^{2010}$ are 606 cifre $\dots\dots\dots 1 \text{ p}$

\Rightarrow prima cifra a lui 2^{2010} este 1 $\dots\dots\dots 1 \text{ p}$

$\lg 5 = 1 - \lg 2 = 0,699 \dots\dots\dots 1 \text{ p}$

$\lg 5^{2009} = 2009 \lg 5 = 1404,29.. \Rightarrow 5^{2009}$ are 1405 cifre $\dots\dots\dots 1 \text{ p}$

$\lg 5^{2010} = 2010 \lg 5 = 1404,99 \Rightarrow 5^{2010}$ are 1405 cifre $\dots\dots\dots 1 \text{ p}$

\Rightarrow prima cifra a lui 5^{2009} este 1 $\dots\dots\dots 1 \text{ p}$