CONCURSUL NAȚIONAL DE MATEMATICĂ APLICATĂ
"ADOLF HAIMOVICI"
etapa locală –9 februarie 2013 -
CLASA A X-A
Filiera teoretică – Profilul real – Specializarea Ştiințe ale naturii

BAREM DE NOTARE ŞI CORECTARE

SUBIECTUL I

a)	$\left(3^{\frac{1}{2}}+3^{-\frac{1}{2}}\right)^2=3+2\cdot 3^{\frac{1}{2}}\cdot 3^{-\frac{1}{2}}+3^{-1}=\frac{16}{3}$	
	$\left(3^{\frac{1}{2}}-3^{-\frac{1}{2}}\right)^2=3-2\cdot 3^{\frac{1}{2}}\cdot 3^{-\frac{1}{2}}+3^{-1}=\frac{4}{3}$	**1p**
	$\left(3^2-3^{-2}\right)^2-\left(3^2+3^{-2}\right)^2=\cancel{3^4}-2\cdot 3^2\cdot 3^{-2}+\cancel{3^{-4}}\cancel{-3^4}-2\cdot 3^2\cdot 3^{-2}\cancel{-3^{-4}}=-4$	**1p**
	$y=6\cdot\frac{20}{-12}=-10$	**1p**
b)	$x=2013^{\frac{a-b}{ab}}\cdot 2013^{\frac{b-c}{bc}}\cdot 2013^{\frac{c-a}{ca}}=2013^{\frac{a-b}{ab}+\frac{b-c}{bc}+\frac{c-a}{ca}}=2013^0=1$	**3p**
c)	$(x-y-10)^{2013}-(-x+y+10)^{2+0+1+3}=1^{2013}-(-1)^6=0$	**1p**

SUBIECTUL II

1.	$a=3\log_{\frac{1}{3}}9\sqrt[3]{3}=3\log_{3^{-1}}3^2\cdot 3^{\frac{1}{3}}=-3\cdot\frac{7}{3}=-7$	**1p**
	$b=9^{\log_3\sqrt{16}}=3^{2\log_3\sqrt{16}}=16$, $c=-4\cdot\left(\frac{1}{4}\right)^{\log_{\frac{1}{4}}9+1}=-4\cdot\left(\frac{1}{4}\right)^{\log_{\frac{1}{4}}\frac{9}{4}}=-9$	**1p**
	Ecuația devine $-7x^2+16x-9=0\Leftrightarrow x\in\left\{1,\frac{9}{7}\right\}$	**1p**
2.		**1p**
	$\frac{\lg x}{y-z}=\frac{\lg y}{z-x}=\frac{\lg z}{x-y}=k\Leftrightarrow \lg x=k(y-z),\ \lg y=k(z-x),\ \lg z=k(x-y)$ (*) Logaritmând $x^x\cdot y^y\cdot z^z=1\Rightarrow x\lg x+y\lg y+z\lg z=0$ şi înlocuind (*) relația este verificată.	**1p**
3.	$S=\left(\frac{1}{2}\cdot\frac{1}{\log_5^2 2}+\frac{1}{2\cdot 3}\cdot\frac{1}{\log_5^2 2}+\ldots+\frac{1}{(n-1)\cdot n}\cdot\frac{1}{\log_5^2 2}\right)\cdot\log^2{}_5 2=$	**1p**
	$\frac{1}{2}+\frac{1}{2\cdot 3}+\ldots+\frac{1}{(n-1)\cdot n}=1-\frac{1}{2}+\frac{1}{2}-\frac{1}{3}\ldots+\frac{1}{(n-1)}-\frac{1}{n}=\frac{n-1}{n}$;	**1p**
	$S=\frac{2012}{2013}\Rightarrow n=2013$.	

SUBIECTUL III

a)	$z_1 = \sqrt{2}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)$	**2p**
b)	$z_2 = \frac{(-9i-1)(-5i-4)}{(-5i+4)(-5i-4)} = \frac{-41+41i}{-41} = 1-i$	**2p**
c)	$(4-5i)z^4 + 1 + 9i = 0 \Leftrightarrow z^4 = \frac{-9i-1}{-5i+4} \overset{b}{\Rightarrow} z^4 = 1-i \overset{a}{\Rightarrow} z^4 = \sqrt{2}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)$ $z_k = \sqrt[4]{\sqrt{2}}\left(\cos\frac{\frac{7\pi}{4}+2k\pi}{4} + i\sin\frac{\frac{7\pi}{4}+2k\pi}{4}\right),\ k \in \{0,1,2,3,4\}$	**1p** **2p**

SUBIECTUL IV

a)	$z^2 = \left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)^2 = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2} = i$ $z^{16} = \left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)^{2016} = \cos 504\pi + i\sin 504\pi = 1$	**1p** **1p**
b)	$S_1 + iS_2 = \cos\frac{\pi}{4} + \cos\frac{2\pi}{4} + \ldots + \cos\frac{2016\pi}{4} + i\left(\sin\frac{\pi}{4} + \sin\frac{2\pi}{4} + \ldots + \sin\frac{2016\pi}{4}\right) =$ $\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right) + \left(\cos\frac{2\pi}{4} + i\sin\frac{2\pi}{4}\right) + \ldots + \left(\cos\frac{2016\pi}{4} + i\sin\frac{2016\pi}{4}\right) =$ $= z + z^2 + \ldots + z^{2016} = \frac{z\left(z^{2016}-1\right)}{z-1} = 0$	**2p** **3p**