

CONCURSUL DE MATEMATICĂ APLICATĂ, „ADOLF HAIMOVICI”

Etapa locală – Constanța, 16.02.2013

filiera tehnologică : profil tehnic, toate specializărilefiliera tehnologică: profil servicii, specializarea resurse naturale și protecția mediului

Clasa a X-a

Barem de corectare și notare

Subiectul 1

$$\text{a) } \log_{54} 3 = \frac{1}{\log_3 54} = \frac{1}{\log_3 27 + \log_3 2} = \frac{1}{3+a} \dots\dots\dots (3\text{p})$$

$$\text{b) } \text{Conditii: } x - 2y > 0, x > 0, y > 0 \dots\dots\dots (1\text{p})$$

$$\lg(x - 2y)^2 = \lg xy \dots\dots\dots (1\text{p})$$

$$(x - 2y)^2 = xy \Rightarrow x^2 - 5xy + 4y^2 = 0 \Rightarrow \left(\frac{x}{y}\right)^2 - 5\left(\frac{x}{y}\right) + 4 = 0, \frac{x}{y} = t \dots\dots\dots (1\text{p})$$

$$t^2 - 5t + 4 = 0 \Rightarrow t_1 = 1 \Rightarrow \frac{x}{y} = 1 \text{ nu convine; } t_2 = 4 \Rightarrow \frac{x}{y} = 4 \dots\dots\dots (1\text{p})$$

Subiectul 2

$$\text{a) } z = 1 + 3i - 3 - i + 1 - 3i - 3 + i = -4 \dots\dots\dots (1\text{p})$$

$$|z| = 4 \dots\dots\dots (1\text{p})$$

$$\text{b). } z' \in R \Leftrightarrow z' = \bar{z}' \Leftrightarrow \dots\dots\dots (1\text{p})$$

$$\Leftrightarrow \frac{z^2 + z + 1}{z^2 - z + 1} = \frac{\bar{z}^2 + \bar{z} + 1}{\bar{z}^2 - \bar{z} + 1} \Leftrightarrow \dots\dots\dots (1\text{p})$$

$$\Leftrightarrow (z - \bar{z})(|z^2| - 1) = 0 \dots\dots\dots (1\text{p})$$

$$z \in C - R \Rightarrow z \neq \bar{z} \Rightarrow \dots\dots\dots (1\text{p})$$

$$\Rightarrow |z| = 1 \dots\dots\dots (1\text{p})$$

Subiectul 3

$$(\sqrt{3a - 2} - 2)(b - 3) + 0 \dots\dots\dots (2\text{p})$$

$$a = 2, b = 3 \dots\dots\dots (2\text{p})$$

$$2 - \sqrt{2 + \sqrt{3}} = \frac{1}{(2 + \sqrt{3})(2 + \sqrt{2 + \sqrt{3}})} \Leftrightarrow \dots\dots\dots (1\text{p})$$

$$\Leftrightarrow (4 - 2 - \sqrt{3})(2 + \sqrt{3}) = 1 \Leftrightarrow \dots\dots\dots (1\text{p})$$

$$\Leftrightarrow (2 - \sqrt{3})(2 + \sqrt{3}) = 1(A) \dots\dots\dots (1\text{p})$$

Subiectul 4

$$\text{a) } 2^x = 31; 31^y = 64 \Rightarrow 31^y = (2^x)^y = 2^{xy} = 64 \dots\dots\dots (2\text{p})$$

$$2^{xy} = 2^6 \Rightarrow xy = 6 \dots\dots\dots (1\text{p})$$

$$\text{b) } a = \frac{1}{\log_5 x^4} = \frac{1}{4 \log_5 x} \dots\dots\dots (2\text{p})$$

$$b = \frac{\log_5 125}{\log_5 x} = \frac{3}{\log_5 x} \dots\dots\dots (1\text{p})$$

$$\Rightarrow \frac{a}{b} = \frac{1}{12} \dots\dots\dots (1\text{p})$$