

1) $|2x - 1| \leq 7 \Leftrightarrow -7 \leq 2x - 1 \leq 7$ (2p)

$-6 \leq 2x \leq 8$ (2p)

$-3 \leq x \leq 4$ (2p)

$M = \{0, 1, 2, 3, 4\}$ (1p)

2) a) $f(-1) = 2; f(2) = -3 \Rightarrow a = -\frac{5}{3}, b = \frac{1}{3}$ (2p)

$f(\sqrt{3}) = -\sqrt{3}; f(-\pi) = \pi \Rightarrow m = -1, n = 0$ (2p)

b) $(f \circ f)(x) = \begin{cases} \frac{25}{9}x - \frac{2}{9}, & x \in Q \\ x, & x \in R - Q \end{cases}$ (3p)

3) a) $a_2 = \frac{3}{5} \cdot \frac{1}{5} = \frac{3}{25}, a_3 = \frac{4}{10} \cdot \frac{3}{25} = \frac{6}{125}, a_4 = \frac{5}{15} \cdot \frac{6}{125} = \frac{2}{125}$ (2p)

b) $a_2 = \frac{3}{5 \cdot 1} a_1, a_3 = \frac{4}{5 \cdot 2} a_2, \dots, a_{n+1} = \frac{n+2}{5 \cdot n} a_n$. Prin înmulțirea relațiilor se obține:

$a_2 \cdot a_3 \cdot \dots \cdot a_{n+1} = \frac{3 \cdot 4 \cdot \dots \cdot (n+1)(n+2)}{5^n \cdot 2 \cdot 3 \cdot \dots \cdot n} a_1 \cdot a_2 \cdot \dots \cdot a_n$ (2p)

$\Rightarrow a_n = \frac{n(n+1)}{2 \cdot 5^n}$ (1p)

c) $b_n = \frac{1}{5^n}$.

$\frac{b_{n+1}}{b_n} = \frac{1}{5} = ct \Rightarrow (b_n)$ progresie geometrică. (2p)

4) a) $\overrightarrow{DE} = \overrightarrow{DB} + \overrightarrow{BE} = \frac{1}{3}\overrightarrow{AB} + \frac{1}{2}\overrightarrow{BC} = \frac{1}{2}\overrightarrow{BC} - \frac{1}{3}\overrightarrow{BA}$ (3p)

b) $\overrightarrow{EF} = \overrightarrow{EC} + \overrightarrow{CF} = \frac{1}{2}\overrightarrow{BC} + \overrightarrow{AB} + \overrightarrow{BC} = \frac{3}{2}\overrightarrow{BC} - \overrightarrow{BA}$ (3p)

c) $\overrightarrow{EF} = 3 \cdot \overrightarrow{DE} \Rightarrow D, E, F$ coliniare (1p)

