## CLASA A IX-A

**Subiectul 1.** $\{y\} = 0,2$ **(1p)**; $[x] + [y] = 4$ **(1p)**; $[2x] + [y] = 5$ **(1p)**; $\left[x + \frac{1}{2}\right] = 1$ **(1p)**, deci $\frac{1}{2} \leq x < \frac{3}{2}$ **(1p)**. Cazul 1. $[x] = 0 \Rightarrow [y] = 4 \Rightarrow y = 4,2$, apoi $x = 0,65$ **(1p)**; Cazul 2. $[x] = 1 \Rightarrow [y] = 3 \Rightarrow y = 3,2$, apoi $x = 1,15$ **(1p)**.

**Subiectul 2.** Verificare $n = 2$ **(2p)**; scrie $P(n+1)$ **(2p)**; Finalizare **(3p)**.

**Subiectul 3.** $a^2 + (a+r)^2 + (a+2r)^2 + (a+3r)^2 = (a+4r)(a+5r)$ **(1p)** $\Rightarrow (a-r)(a+2r) = 0$ **(2p)**. Cazul $a = r$, numerele sunt $r, 2r, 3r, 4r, 5r, 6r$ **(1p)**; $r^2 + (2r)^2 + (3r)^2 + (4r)^2 = r \cdot 2r \cdot 3r \Rightarrow r = 0$ sau $r = 5$ **(1p)**. Cazul $a = -2r$, numerele sunt $-2r$, $-r$, $0$, $r$, $2r$, $3r$ **(1p)**. $(-2r)^2 + (-r)^2 + 0^2 + r^2 = (-2r)(-r)0 \Rightarrow r = 0$ **(1p)**

**Subiectul 4.** a) $\frac{DF}{FE} = \frac{BM}{MC} \cdot \frac{AC}{AB} \cdot \frac{AD}{AE}$ **(2p)**; $\frac{DF}{FE} = \frac{n}{m}$ **(1p)**; b) Exprima $\overrightarrow{AF}$ in functie de $\overrightarrow{AD}$ si $\overrightarrow{AE}$ **(2p)**; Finalizare **(2p)**.
**Nota:** pentru rezolvarea punctului b), folosind a) (nedemonstrat) se vor acorda (3p).

## CLASA A X-A

**Subiectul 1.** $f(f...f(x)) + f(f...f(-x)) = 2b(a^{n-1} + a^{n-2} + ... + 1)$ **(2p)**; $b = 0$ este solutie **(1p)**; $a^{n-1} + a^{n-2} + ... + 1 = 0 \Rightarrow a^n = 1$ **(2p)**; daca $n$ impar, nu sunt solutii pentru $a$ **(1p)**; daca $n$ par, $a = -1$ **(1p)**.

**Subiectul 2.** $x \leq -2 \Rightarrow 2^{x+1} \leq \frac{1}{2} < x^2 + x + 1$ **(2p)**; $x = -1$ solutie **(1p)**; afirma ca pentru $x \geq 1$, avem: $2^{x+1} > x^2 + x + 1$ **(1p)**; verificare $x = 1$ **(1p)**; demonstratia corecta a pasului de inductie **(2p)**.

**Subiectul 3.** i) $1 + a + b = 0$ **(1p)**; $4 + 2a + b = 0$ **(1p)**; $a = -3$, $b = 2$ **(1p)**. ii) pune $z = x + iy$ in $z^2 - 3|z| + 2 = 0$ : $(x+iy)^2 - 3\sqrt{x^2+y^2} + 2 = 0$ **(1p)**; $\begin{cases} x^2 - y^2 - 3\sqrt{x^2+y^2} + 2 = 0 \\ xy = 0 \end{cases}$ **(1p)**; Cazul $x = 0$ **(1p)**; Cazul $y = 0$ **(1p)**.

**Subiectul 4.** $a^{3/2} = b$, $c^{5/4} = d$ **(1p)**; $a^3 = b^2$, $c^5 = d^4$ **(1p)**; $a = x^2$, $b = x^3$ **(1p)**; $c = y^4$, $d = y^5$ **(1p)**; $a - c = x^2 - y^4 = 9$ **(1p)**; $(x - y^2)(x + y^2) = 9 \Rightarrow x = 5$, $y = 2$ **(1p)**; $b - d = x^3 - y^5 = 93$ **(1p)**.

## CLASA A XI-A

**Subiectul 1.** $a_n$ descrescator **(2p)**; $a_n$ marginit **(1p)**; $\lim a_n = 1$ **(1p)**; $b_n = \frac{2(n+1)}{n+2}$ **(2p)**; $\lim b_n = 2$ **(1p)**.

**Subiectul 2.** $x = t^{12}$ **(1p)**; $\lim_{t\to 1} \frac{\alpha\left(t^6-1\right)+\beta\left(t^4-1\right)+\gamma\left(t^3-1\right)}{(t-1)^3(t^{11}+...+t+1)^3}$ **(1p)**; $\frac{1}{12^3}\lim_{t\to 1} \frac{\alpha\left(t^6-1\right)+\beta\left(t^4-1\right)+\gamma\left(t^3-1\right)}{(t-1)^3}$ **(1p)**; $\lim_{t\to 1} \frac{\alpha\frac{t^6-1}{t-1}+\beta\frac{t^4-1}{t-1}+\gamma\frac{t^3-1}{t-1}}{(t-1)^2}$ **(1p)**; Trebuie ca $6\alpha + 4\beta + 3\gamma = 0$ **(1p)**; $\lim_{t\to 1} \frac{\alpha\left[t^5+...+1\right]+\beta\left[t^3+...+1\right]+\gamma\left[t^2+t+1\right]}{(t-1)^2}$
$= \lim_{t\to 1} \frac{\alpha\left[\left(t^5-1\right)+...+(t-1)\right]+\beta\left[\left(t^3-1\right)+...+(t-1)\right]+\gamma\left[\left(t^2-1\right)+(t-1)\right]}{(t-1)^2}$ **(1p)**;
$\lim_{t\to 1} \frac{1}{t-1}\left\{\alpha\left[\frac{t^5-1}{t-1}+...+1\right]+\beta\left[\frac{t^3-1}{t-1}+...+1\right]+\gamma\left[\frac{t^2-1}{t-1}+1\right]\right\}$, deci trebuie si $(5+4+3+2+1)\,\alpha+(3+2+1)\,\beta+(2+1)\,\gamma = 15\alpha+6\beta+3\gamma = 0$ **(1p)**.

**Subiectul 3.** $X_1^2 - X_2^2 = B\,(X_1 - X_2)$ **(1p)**; $(X_1 + X_2)\,(X_1 - X_2) = B\,(X_1 - X_2)$ **(1p)**; $X_1 + X_2 = B$ **(2p)**; $C = BX_1 - X_1^2 = (X_1 + X_2)\,X_1 - X_1^2 = X_1X_2$ **(3p)**.

**Subiectul 4.** $\mathrm{tr}(AB) = 2014$ **(1p)**; $\det(AB) = -1$ **(1p)**; $\det(BA) = \det(AB) = -1 \neq 0$, deci $A$, $B$, $BA$ inversabile **(1p)**; Cayley: $(AB)^2 - 2014\,(AB) - I_2 = 0_2$ **(1p)**; $A^{-1}\left[(AB)^2 - 2014\,(AB) - I_2\right]B^{-1} = 0_2$ **(1p)**; $BA - 2014I_2 - A^{-1}B^{-1} = 0_2$ **(1p)** si $A^{-1}B^{-1} = (BA)^{-1}$ **(1p)**.

## CLASA A XII-A

**Subiectul 1.** $H \cap aH = \emptyset$ **(3p)**; $K$ parte stabila **(2p)**; existenta simetricului **(2p)**.

**Subiectul 2.** $\frac{f(x)-f(k)}{x-k} = a_2x + a_1 + ka_2$ **(3p)**; $F\,(x) = \frac{1}{2}a_2x^2 + (a_1 + ka_2)\,x + a_0$ **(3p)**; finalizare **(1p)**.

**Subiectul 3.** $\frac{1}{n}\sum_{k=1}^{n}\sqrt{1-\left(\frac{k}{n}\right)^2}$ **(1p)**; $\int_0^1 \sqrt{1-x^2}dx$ **(1p)**; $= \pi/4$ **(2p)**; $\frac{1}{n}\sum_{k=1}^{n}\frac{1}{1+\left(\frac{k}{n}\right)^2}$ **(1p)**; $\int_0^1 \frac{1}{1+x^2}dx$ **(1p)**; $= \pi/4$ **(1p)**

**Subiectul 4.** $\int_0^1 = \int_0^{1/3} + \int_{1/3}^{1/2} + \int_{1/2}^{2/3} + \int_{2/3}^{1}$ **(3p)**; pentru calculul corect al acestor integrale se acorda cate **(1p)**.