Barem clasa a VIII-a
(OLM 2014-etapa locală)

Of. 10 p

Subiectul I.

Pornim de la inegalitatea evidentă: $\dfrac{a+b}{\sqrt{ab}} \geq 2, (\forall), a, b \in N^*$, avem: **(10p)**

$$\frac{3}{\sqrt{2}}+\frac{5}{\sqrt{6}}+\frac{7}{\sqrt{12}}+\ldots+\frac{2n-1}{\sqrt{n^2-n}}+\frac{2n+1}{\sqrt{n^2+n}} \geq 2\cdot\left(\frac{2n+2}{2}-1\right)=2\cdot(n+1-1)=2n$$ **(10p)**

Subiectul II.

Relația din enunț devine: $x^2-2\sqrt{3}x+3+y^2+4y+4=4 \Leftrightarrow (x-\sqrt{3})^2+(y+2)^2=4$. **(5p)**

Atunci $(x-\sqrt{3})^2 \leq 4$ si $(y+2)^2 \leq 4$.

$(x-\sqrt{3})^2 \leq 4 \Leftrightarrow |x-\sqrt{3}| \leq 2 \Leftrightarrow x \in [-2+\sqrt{3}, 2+\sqrt{3}] = S_1$. **(5p)**

Dacă $(y+2)^2 \leq 4$ obținem $-2 \leq y+2 \leq 2 \Leftrightarrow y \in [-4,0] = S_2$. **(5p)**

Din $S_1 \cap S_2 = [-2+\sqrt{3}, 0]$ putem avea $[x]=[y]=-1$ sau $[x]=[y]=0$

Dacă $[x]=[y]=-1$ obținem $x \in [-2+\sqrt{3}, 0)$ *și* $y \in [-1,0)$.

Dacă $[x]=[y]=0$ atunci $x \in [0,1)$ *şi* $y=0$. **(5p)**

Subiectul III.

a) $(a+4)(2a+12)-16=2(a+2)(a+8)$; $a(a+1)-2=(a-1)(a+2)$ $\Rightarrow a \in Z \setminus \{-8,-2,1\}$ **(5p)**

b) $\dfrac{1}{E_1}+\dfrac{1}{E_2}=\dfrac{2a+7}{a+2} \in N \Rightarrow$ $a \in \{-1,-5,1\}$ **(10p)**

c) $a=-3$ **(5p)**

Subiectul IV.

Desen corect **(5p)**

a) $d\left(A,(DBC)\right)=\dfrac{12}{13}$; **(CU JUSTIFICĂRI)** **(10p)**

b) $A_{DAB}=1,5m^2, A_{DAC}=2m^2, A_{DBC}=6,5m^2, A_{ABC}=6m^2$ **(5p)**

c) $\sin\left(\sphericalangle((DBC);(ABC))\right)=\dfrac{5}{13}$ **(CU JUSTIFICĂRI)** **(10p)**