Barem de corectare OML Clasa a XI- a, 2014

1. $\lim_{n\to\infty}\sin\left(\pi\sqrt{4n^2+2n+3}\right)=\lim_{n\to\infty}\sin\left(\pi\sqrt{4n^2+2n+3}-2n\pi\right)$..................2p

   $=\lim_{n\to\infty}\frac{(2n+3)\pi}{n\left(\sqrt{4n^2+2n+3}+2\right)}$..2p

   Finalizare..3p

2. a) Dă exemplu de matrici cu proprietatea cerută..3p

   b) Dacă $A,B\in M_2(R)$ cu $AB=BA$ at $\det(A^2+B^2)=\det(A+iB)(A-iB)$ 1p

   $=|\det(A+iB)|^2\geq 0$..1p

   $\det\begin{pmatrix}2 & 3\\ 3 & 2\end{pmatrix}=-5$ 1p

   Finalizare..1p

3 a) $\lim_{x\to 0}\frac{(tg(a_1x)+\cdots+tg(a_nx))^2}{x\left(tg(a_1^2x)+\cdots tg(a_n^2x)\right)}=\lim_{x\to 0}\frac{x^2\left(\frac{tg(a_1x)}{a_1x}a_1+\cdots+\frac{tg(a_nx)}{a_nx}\right)^2}{x^2\left(\frac{tga_1^2x}{a_1^2}a_1^2+\cdots+\frac{tg(a_n^2x)}{a_n^2}a_n^2\right)}$ 2p

$=\frac{(a_1+\cdots+a_n)^2}{a_1^2+\cdots+a_n^2}$..2p

b) $b_n=\frac{(a_1+a_2+\cdots+a_n)^2}{a_1^2+\cdots+a_n^2}\geq n$ deci $n(a_1^2+a_2^2+\cdots+a_n^2)\leq(a_1+\cdots+a_n)^2$ 1p

Dar $(a_1+\cdots+a_n)^2\leq(a_1^2+\cdots+a_n^2)^2n$ 1p

Finalizare.. 1p

4. Fie $X=\begin{pmatrix}x & y\\ z & t\end{pmatrix}$

   i) Calculează AX 1p

   Calculează X..1p

   Finalizare 1p

   ii) $X^3=\begin{pmatrix}2 & 1\\ 1 & 2\end{pmatrix}$ Atunci $X^4=\begin{pmatrix}2 & 1\\ 1 & 2\end{pmatrix}X$ și $X^4=X\begin{pmatrix}2 & 1\\ 1 & 2\end{pmatrix}$

   Deci $\begin{pmatrix}2 & 1\\ 1 & 2\end{pmatrix}X=X\begin{pmatrix}2 & 1\\ 1 & 2\end{pmatrix}$ și $X=\begin{pmatrix}u & v\\ v & u\end{pmatrix}$..1p

   Calculează X^3 ....1p

   Finalizare 2p