OLIMPIADA DE MATEMATICĂ
ETAPA LOCALĂ
14.02.2015

CLASA A VIII-A

Problema 1.

Problema1	Etapa de rezolvare	Punctaj
a)	$(x+y)^2 \geq 9$	1
	$x^2+y^2 \geq \frac{9}{2}$	2
	$2x+2y \geq 6$	1
	$(x+1)^2+(y+1)^2 \geq \frac{9}{2}+6+2 > 12$	1
b)	Folosind inegalitatea: $2x^2+2y^2 \geq (x+y)^2$	
	$(x-1)^4+(y+2)^4 \geq \frac{1}{2}[(x-1)^2+(y+2)^2]^2$	1
	$\geq \frac{1}{2}\cdot\frac{1}{4}(x-1+y+2)^4 \geq \frac{1}{8}\cdot 4^4 = 32$	1

Problema 2.

Problema2	Etapa de rezolvare	Punctaj
a)	$\sqrt{x} = r - \sqrt{y}$	1
	$x = r^2 - 2r\sqrt{y} + y$	1
	$\sqrt{y} = \frac{r^2+y-x}{2r} \in Q$	2
b)	Aducerea la forma $\sqrt{x}+\sqrt{y}=7$	1
	Conform a) rezultă $\sqrt{x},\sqrt{y} \in N$	1
	(x,y)$\in \{(1,36),(4,25),(9,16),(16,9),(25,4),(36,1)\}$	1

Problema 3.

Problema3	Etapa de rezolvare	Punctaj
	Conform teoremei bisectoarei in ABV avem $\frac{VX}{XB} = \frac{VA}{AB}$	1
→	$\frac{VA}{AB} = \frac{VB}{BC} = \frac{VC}{CA} \rightarrow \frac{VX}{XB} = \frac{VY}{YC} = \frac{VZ}{ZA}$	2
	XY ∥ BC si XZ ∥ AB → (ABC) ∥ (XYZ)	1
←	(ABC) ∥ (XYZ) → $\frac{VA}{AB} = \frac{VB}{BC} = \frac{VC}{CA}$	2
	AB=BC=CA → VA=VB=BC → piramida regulata	1

Problema 4.

Problema4	Etapa de rezolvare	Punctaj
a)	AT ⊥ BC, AT= $\sqrt{3} + 1$	2
	d(V, BC)= VT= $(\sqrt{3} + 1)\sqrt{2}$	1
b)	AB = BC sin 15= $2\sqrt{2}$	1
	AU ⊥ VB deci d(AC, VB)= AU	2
	AU=$\frac{VA \cdot AB}{VB}$, calcul algebric	1